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TITLE: NUCLEON-NUCLEON CORRELATIONS DETECTED VIA PION DOUBLE-CHARGE-EXCHANGE REACTIONS

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NUCLEON-NUCLEON CORRELATIONS DETECTED VIA PION DOUBLE-CHARGE-EXCHANGE REACTIONS†

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ABSTRACT

Double-charge-exchange reactions induced by pions are shown to furnish a direct means of detecting nucleon-nucleon correlations in nuclei. For the particular case of double isobaric analog transitions the effect of correlations is easily isolated and quite strong. Examination of the properties of the position correlations implicit in the nuclear shell model provides an explanation of several experimentally observed features of double-charge-exchange reactions, including in particular, their dependence on the number of valence neutrons present. An explicit demonstration of the role played by nucleon-nucleon correlations is presented for the double-charge-exchange cross-sections of the isotopes ^{42}Ca, ^{44}Ca and ^{48}Ca, recently measured at LAMPF. Our analysis explains why the cross section for the double-charge-exchange reaction on ^{48}Ca, in which 28 neutron pairs participate, is comparable and indeed smaller than the corresponding cross-section for ^{42}Ca with its single valence neutron pair. It also predicts the variation of angular distributions over the family of isotopes.

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INTRODUCTION

Although the existence of position correlations between nuclear particles is an essential part of most models of the nucleus, the experimental attempts to isolate their signatures have had a rather frustrating history. Other effects tended to dominate them quantitatively, and usually those effects have been too poorly defined to be separable in any practical way. Double scattering amplitudes, for example, are sensitive to correlations, but cannot easily be separated from the typically much larger amplitudes for single scattering. Pion absorption processes are certainly sensitive to correlation effects but their mechanisms and the number of nucleons involved is not yet well determined. Double-charge-exchange processes of pions in nuclei, however, do not suffer from these drawbacks; they have a cleaner mechanism and can be described by a double-collision amplitude that is not hidden by any interfering single-collision amplitude. In particular those double charge exchanges that take place between isobaric analog states are quite similar to elastic double scattering processes, and can thus be analyzed in considerable detail. This is particularly the case for low energy reactions, e. g. for pion kinetic energies $\simeq$ 35 - 50 MeV, since the πN interaction is relatively weak at these energies. Indeed, an approach based on the lowest (second) order term in the coherent multiple-scattering expansion was shown recently[1] to describe such processes quite accurately. The analysis of ref. 1 focused on the double-charge-exchange process at 50 MeV in ^{14}C, a target nucleus with two valence neutrons that the reaction transforms into protons. The magnitude and the shape of the differential cross-section were found to be quite responsive to the position correlations of these two valence neutrons.

The purpose of this paper is to describe a generalization of the approach of ref. 1, which will enable us to describe double-charge-exchange reactions on targets with more than two valence neutrons. Our work has been largely stimulated by the recent measurements[2, 3, 4] of differential cross-sections for double isobaric analog transitions in the calcium isotopes ^{42}Ca, ^{44}Ca, and ^{48}Ca. The measurements have presented an interesting puzzle in the smallness of the magnitudes of the differential cross-section for ^{48}Ca relative to that for ^{42}Ca. These experimental results permit us now to give a much more detailed and explicit demonstration of the role played by nucleon-nucleon correlations. The results of our analysis described in this presentation have also been reported earlier.[5, 6]

TRANSITION AMPLITUDE FOR DOUBLE-CHARGE-EXCHANGE REACTION

The charge-exchange processes we study are induced by positive pions incident on the nucleus (N, Z) and lead to the isobaric analog state in the nucleus $(N-2, Z+2)$. Such analog-state transitions are essentially elastic. The nucleon isospins are simply rotated, while the configuration-space and spin wave functions of the nucleons undergo no change. The nucleus (N, Z) will be pictured as consisting of a closed-shell core plus $N-Z$ valence neutrons. At the low energy of 35 MeV the nucleus is fairly transparent to pions. We have shown in our

study[1] of double charge exchange in ^{14}C at the somewhat higher energy of 50 MeV that no appreciable error is introduced by using the second-order impulse approximation to describe the double-collision process. At 35 MeV that approximation should be comparably good and the integrated background effects of nuclear elastic scattering and absorption small.[1] Within this approximation double charge exchange is simply a sequence of two single exchanges. In the double analog transition neither of these exchanges can take place on nucleons in the nuclear core since such processes lead to final states outside the isobaric multiplet. The nuclear core therefore plays a passive role, and the matrix element of the $\pi^+ \rightarrow \pi^-$ process can be expressed simply as a sum of contributions of pairs of valence nucleons.

Our expression for the transition amplitude for the double-charge-exchange reaction $\pi^+(N-2, Z+2, 0^+) \rightarrow \pi^0(N, Z, 0^+)$ in the second-order impulse approximation is given by the matrix element

$$F_{DCX}(\mathbf{Q}) = (\pi^-(N-2, Z+2, 0^+) \mid \sum_{m \neq n}^{N-Z} \mathcal{T}(\mathbf{Q}, \mathbf{r}_m, \mathbf{r}_n) \mid \pi^+(N, Z, 0^+)) \quad , \quad (1)$$

where $\mathcal{T}$ are appropriate transition operators dependent on the coordinates, spins, and isospins of two nucleons, $\mathbf{Q} = \mathbf{k}_f - \mathbf{k}_i$ is the momentum transferred by the pions, $\mathbf{k}_i$ and $\mathbf{k}_f$ being the incident and scattered pion momenta respectively.

The transition operator $\mathcal{T}$ for the pair of nucleons located at $\mathbf{r}_m$ and $\mathbf{r}_n$ expressed in terms of the spin dependent charge-exchange amplitude of the n-th nucleon a by $F_n(\mathbf{k}_f, \mathbf{k}_i)$, can be written as

$$\mathcal{T}(\mathbf{Q}, \mathbf{r}_m, \mathbf{r}_n) = -\frac{1}{2\pi^2} \int [F_m(\mathbf{k}_f, \mathbf{p}) G(p) F_n(\mathbf{p}, \mathbf{k}_i) + F_n(\mathbf{k}_f, \mathbf{p}) G(p) F_m(\mathbf{p}, \mathbf{k}_i)] d\mathbf{p}. \quad (2)$$

The integration variable $\mathbf{p}$ is the momentum of the neutral pion in its states that intervene between the two collisions to the other, and

$$G(p) = (k^2 - p^2 + i\epsilon)^{-1} \quad (3)$$

is the complex pion propagator. We note that the transition amplitudes in the expression (2) have been summed over all possible intermediate nuclear states by means of the closure approximation. Had we restricted the summation to the intermediate analog state, we would, in fact, have excluded altogether the possibility of treating correlations.[1] It may be worth pausing for a moment to emphasize that point.

There is a rather appealing but flawed argument sometimes given to support the contrary idea – that for the double analog transitions we are considering, the only intermediate state that contributes to the observed amplitude is the analog state itself. It may be useful to discuss that argument as a cautionary example. The argument maintains that neither of the two charge-exchanging collisions of the pion can in any way alter the space or spin states of either of the two struck neutrons. That should be true because the final nuclear state, as an analog of the initial state, must have a space–spin wave function identical to the original one.

Furthermore, since the individual collisions take place with different neutrons, if either neutron wave were changed in either of the two steps of the collision process,there is no way in which that change could be reversed in the other step; the final state would necessarily lie outside the original isospin multiplet. But then if neither struck neutron can change its space-spin state, the intermediate state must also belong to the same isospin multiplet, and no contribution to the process would seem to come from any of the other possible intermediate states. What, now, is wrong with this picture?

The problem is a fairly fundamental one. The argument we've just noted assumes a bit too loosely that there is such a thing as the state of each neutron. If we were entitled to use an independent particle model of the nucleus, and could use a product wave function to represent it, then the individual states the argument refers to would be definable, and the intermediate analog state would indeed be the only one to contribute. But in a system containing nucleon-nucleon correlations the argument breaks down. If the collision of a pion with one of two correlated nucleons alters the wave function of one of them, it inevitably alters the wave function of both. That fact breaches the constraint noted earlier. The identity of the initial and final nuclear states no longer implies that the intermediate state must be the same.

The operative principle, in effect, is the same as that underlying the classic quantum mechanical paradox of Einstein, Podolsky, and Rosen.[7] When particles are in a correlated state, their states can no longer be discussed individually. Making a measurement on one inevitably influences the other, wherever it may be, miles away or just elsewhere in the same nucleus.

The amplitude F_n that describes an individual charge-exchange process on the n-th nucleon contains the familiar retardation phase factor $e^{i(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r}_n}$. It also depends upon the nucleon spin $\vec{\sigma}_n$, its isospin $\vec{\tau}_n$ and the pion isospin $\mathbf{T}_\pi$. It can be written as

$$\begin{aligned} F_n(\mathbf{p}',\mathbf{p}) &= (\vec{\tau}_n \cdot \mathbf{T}_\pi) e^{i(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r}_n} M_n(\mathbf{p}',\mathbf{p}), \\ M_n(\mathbf{p}',\mathbf{p}) &= [f(\mathbf{p}',\mathbf{p}) + i\vec{\sigma}_n \cdot (\mathbf{p} \times \mathbf{p}')g(\mathbf{p}',\mathbf{p})]h(p',p). \end{aligned} \tag{4}$$

In the latter expression $f(\mathbf{p}',\mathbf{p})$ and $g(\mathbf{p}',\mathbf{p})$ are the spin-independent and spin-flip charge-exchange amplitudes, respectively. The function $h(p',p)$ represents the off-mass-shell form factor. We shall assume for it, as in our previous analysis[1], the frequently used expression[8]

$$h(p,k) = h(k,p) = \frac{\Lambda^2 + k^2}{\Lambda^2 + p^2}, \tag{5}$$

where $k = |\mathbf{k}_i|$, and the parameter Λ may be interpreted as the reciprocal of its range of the πN interaction. The presence of such a form factor prevents the scattering amplitude (4) from having an unphysical and even divergent behavior for large values of pion momenta.

Another, equivalent, way of incorporating the the effect of form factor (5) in the matrix element (2) is to combine it with the free-pion propagator (3) by defining an effective propagator

$$\tilde{G}(p) = G(p)h^2(p,k) \ , \tag{6}$$

which has the following form in the configuration space

$$\tilde{G}(r) = \frac{1}{(2\pi)^3}\int e^{i\mathbf{p}\cdot\mathbf{r}}\tilde{G}(p)d\mathbf{p} = -\frac{e^{ikr}}{4\pi r} + \frac{e^{-\Lambda r}}{4\pi r}\left[1 + \frac{k^2+\Lambda^2}{2\Lambda}r\right] \ , \tag{7}$$

while $G(r)$ is simply the first of these terms. The propagators $G(r)$ and $\tilde{G}(r)$ behave differently only for $r \to 0$, where $G(r)$ becomes infinite while $\tilde{G}(r)$ tends to a finite value. In this respect the effective propagator $\tilde{G}$ can be thought of as an averaged propagator for finite-sized particles. The r dependence of the propagator plays an important role in the matrix element (2); it emphasizes strongly configurations in which the two neutrons are close together. The effective propagator $\tilde{G}(r)$ weakens that emphasis to a degree, but still provides that double charge exchange is most probable when the separations are small.

By evaluating the isospin matrix elements and by summing terms involving all the contributing pairs of nucleons, we obtain the following expression for the double-charge-exchange amplitude

$$F_{DCX}(\mathbf{Q}) = \sqrt{\tfrac{1}{2}(N-Z)(N-Z-1)}M_{N-2}(\mathbf{Q}) \ , \tag{8}$$

where $M_{N-2}(\mathbf{Q})$ is the double-charge-exchange amplitude on one arbitrarily chosen pair, of neutrons, say 1 and 2,

$$M_{N-2}(\mathbf{Q}) = -\frac{1}{2\pi^2}\langle 1,\ldots,N-Z \mid \int e^{i\mathbf{q}_1\cdot\mathbf{r}_1+i\mathbf{q}_2\cdot\mathbf{r}_2}[M_1(\mathbf{k}_f,\mathbf{p})G(p)M_2(\mathbf{p},\mathbf{k}_i)+ \\ + M_2(\mathbf{k}_f,\mathbf{p})G(p)M_1(\mathbf{p},\mathbf{k}_i)]\, d\mathbf{p} \mid 1,\ldots,N-Z\rangle, \tag{9}$$

and $\mathbf{q}_1 = \mathbf{p} - \mathbf{k}_i$ and $\mathbf{q}_2 = \mathbf{k}_f - \mathbf{p}$ are the momenta transferred to the pion in its succesive collisions with two neutrons.

The square of the matrix element (8) contains as a factor the total number of valence neutron pairs. While that number increases, for example, from 1 to 28 in the sequence of isotopes from ^{42}Ca to ^{48}Ca, it would be quite erroneous to assume that the cross section for double-charge-exchange increases in any similar way. In fact the effect of nucleon-nucleon correlations, as we shall show, is to make the matrix element $M_{N-2}(\mathbf{Q})$ decrease in modulus, so that the double-charge-exchange cross-section actually decreases in magnitude as $N-Z$ increases within a given nuclear shell.

It is clear from the structure of Eq. (9) that the dependence of $M_{N-Z}(\mathbf{Q})$ on the nuclear wave functions enters the calculation through the two-particle form factor

$$S_{N-Z}(\mathbf{q}_1,\mathbf{q}_2) = \langle 1,...,N-Z \mid e^{i\mathbf{q}_1\cdot\mathbf{r}_1+i\mathbf{q}_2\cdot\mathbf{r}_2} \mid 1,...,N-Z\rangle$$
$$= \int e^{i\mathbf{q}_1\cdot\mathbf{r}_1+i\mathbf{q}_2\cdot\mathbf{r}_2}\rho^{(2)}_{N-Z}(\mathbf{r}_1,\mathbf{r}_2)d\mathbf{r}_1 d\mathbf{r}_2\ , \qquad (10)$$

where $\rho^{(2)}_{N-Z}(\mathbf{r}_1,\mathbf{r}_2)$ is the two-particle density of the valence neutrons in the initial ground state. The wave function also enters Eq.(8) through an analogous spin-dependent form factor

$$S^{spin}_{N-Z}(\mathbf{q}_1,\mathbf{q}_2) = \langle 1...,N-Z \mid \vec{\sigma}_1\cdot(\hat{\mathbf{k}}_i\times\hat{\mathbf{p}})\vec{\sigma}_2\cdot(\hat{\mathbf{p}}\times\hat{\mathbf{k}}_f)e^{i\mathbf{q}_1\cdot\mathbf{r}_1+i\mathbf{q}_2\cdot\mathbf{r}_2} \mid 1,...,N-Z\rangle, \qquad (11)$$

that describes double-spin-flip processes. The role of these spin-flip transitions in the double-charge-exchange process, although quantitatively significant, seems not to be as important as the effect of spatial correlation in explaining the general behavior of the observed cross-sections. We shall therefore forego the spin-dependent terms explicitly in the present discussion and shall concentrate instead on the effects of spatial correlation, but we do this with the understanding that the results we s ate for our detailed calculations nonetheless include all effects of spin dependence.

If there were no correlations among the valence neutrons, the two-particle density $\rho^{(2)}_{N-Z}(\mathbf{r}_1,\mathbf{r}_2)$ would factorize into the product $\rho(r_1)\rho(r_2)$ of the two single-particle densities, and in that case, as we have shown in ref. 1, the only intermediate nuclear state that would then contribute to the double-charge-exchange process would be the isobaric analog state. We are assuming, on the contrary, that correlations are present and have therefore summed over all accessible intermediate states, both analog and non-analog. To express the effect of correlation more explicitly we introduce the correlation function

$$C_{N-Z}(\mathbf{r}_1,\mathbf{r}_2) = \rho^{(2)}_{N-Z}(\mathbf{r}_1,\mathbf{r}_2) - \rho(r_1)\rho(r_2), \qquad (12)$$

which permits us to write the form factor (10) as the sum of two terms,

$$S_{N-Z}(\mathbf{q}_1,\mathbf{q}_2) = S_0(q_1)S_0(q_2) + \int e^{i\mathbf{q}_1\cdot\mathbf{r}_1+i\mathbf{q}_2\cdot\mathbf{r}_2}C_{N-Z}(\mathbf{r}_1,\mathbf{r}_2)d\mathbf{r}_1 d\mathbf{r}_2. \qquad (13)$$

where

$$S_0(q) = \int e^{i\mathbf{q}\cdot\mathbf{r}}\rho(r)d\mathbf{r} \qquad (14)$$

is the single-particle form factor. The first of the terms in Eq.(13) is associated with transitions that take place through the analog state; the two single-particle form factors it contains tend to be strongly peaked in the forward direction. The second term, which expresses the effect of correlations, is associated with

transitions that take place only through the non-analog states and vanishes for $q_1 = 0$ or $q_2 = 0$. These two terms then will lead, in general, to transition amplitudes having different angular dependences.

SHELL-MODEL CORRELATION EFFECTS IN DOUBLE-CHARGE-EXCHANGE REACTIONS

In the following discussion we examine the effects of the primary sources of the correlations of positions and spins of the valence neutrons which follow from the constraints implicit in nuclear shell structure.

The constraint that the angular momenta of the valence nucleons must sum to zero leads to particularly strong spatial correlation for a single pair, $N - Z = 2$. Additional neutron pairs added to the valence shell tend furthermore to have angular momenta coupled to zero as well. We have applied these constraints, together with the requirement of antisymmetry, to the wave functions for the shell of $j = l + \frac{1}{2}$ orbitals. The single-particle density $\rho(r)$ that results is just the squared radial wave function for the shell and is spherically symmetric. The correlation function, on the other hand, for $N - Z$ even, is

$$C_{N-Z}(\mathbf{r},\mathbf{r}') = \kappa_{N-Z}\rho(r)\rho(r')\left((l+1)[P_l(\hat{\mathbf{r}}\cdot\hat{\mathbf{r}}')]^2 + \frac{1}{l+1}[(P_l^1(\hat{\mathbf{r}}\cdot\hat{\mathbf{r}}')]^2 - 1\right), \tag{15}$$

where the constant κ_{N-Z} is given by

$$\kappa_{N-Z} = \frac{2j+3-2(N-Z)}{(2j-1)(N-Z-1)}, \tag{16}$$

and P_l and P_l^1 are the normal and associated Legendre functions respectively. We note that C_{N-Z} has the same shape for all values of $N - Z$ within the $j = l + \frac{1}{2}$ shell. The coefficient κ_{N-Z} evidently governs the magnitude and sign of the correlation effect. In particular, for a filled shell with $N - Z \to \infty$, we have

$$\lim_{N-Z\to\infty} C_{N-Z}(\mathbf{r},\mathbf{r}) = -\tfrac{1}{2}\rho^2(r), \tag{17}$$

which expresses the correlation characteristic of a Fermi gas.

The complete expression for the spin-independent form factor (10) that corresponds to the correlation function (15) is

$$S_{N-Z}(q_1,q_2) = S_0(q_1)S_0(q_2) + \kappa_{N-Z}\sum_{L\neq 0} a_L S_L(q_1)S_L(q_2)P_L(\hat{\mathbf{q}}_1\cdot\hat{\mathbf{q}}_2), \tag{18}$$

where

$$S_L(q) = \int dr r^2 j_L(qr)R_{nl}^2(r) \tag{19}$$

is the form factor associated with the intermediate nuclear state with angular momentum L, j_L is a spherical Bessel function, and R_{nl} the radial wave

function[9] corresponding to principal quantum number n and orbital momentum l and

$$a_L = (2j+1)(2l+1)^2 \left[\begin{Bmatrix} l & l & L \\ j & j & \frac{1}{2} \end{Bmatrix} C^{L0}_{l0l0} \right]^2 , \tag{20}$$

given it terms of the Wigner 6-j symbol and the Clebsch-Gordon coefficient. We do not present here the lengthier expression for the spin-dependent form factor. We may note, however, that it contains no contribution from the uncorrelated part of the two-particle density. In effect, the double-spin-flip transitions take place only on correlated pairs.

In the course of the double-charge-exchange process a neutral pion must propagate from one of a pair of participating neutrons to the other. Its propagator provides a weighting in the integrand of Eq. (9) that is naturally greatest when the two neutrons have small separation. It is that tendency of the process to favor configurations in which the valence neutrons are close together that gives particular emphasis to the effects of spatial correlations.[1]

SHELL-MODEL CORRELATION EFFECTS IN CALCIUM ISOTOPES

A very instructive insight into the role played by correlations in the transition amplitudes present in the multiple scattering expansion can be gained by examining how the correlations affect the distribution of the relative coordinate of two nucleons on which the reaction takes place. Such a density distribution – separation density function has been used, in fact, in our previous detailed analysis of correlation effects in high energy hadron-nucleus reactions. [10] We note, that, the separation density of present interest is that of the valence neutrons only, and it is, in general quite different from that of all nucleon pairs. In fact, the correlation effects in the valence neutron shell, to which the double-charge-exchange reactions are sensitive to, are different and quantitatively more important than those affecting the elastic double-collision amplitude.

The separation density of the valence neutrons is

$$\rho_{sep}(r) = \int \rho^{(2)}_{N-s}(\mathbf{R} + \tfrac{1}{2}\mathbf{r}, \mathbf{R} - \tfrac{1}{2}\mathbf{r}) d\mathbf{R} = \frac{1}{(2\pi)^3} \int e^{-i\mathbf{q}\cdot\mathbf{r}} S_{N-s}(\mathbf{q}, -\mathbf{q}) d\mathbf{q} \ . \tag{21}$$

It is this function, weighted with the spatial pion propagator and its derivatives, and integrated over $\mathbf{r}$, that governs the magnitude of the forward values ($\mathbf{Q} = 0$) of the double-charge-exchange cross section.

We have used a set of harmonic-oscillator-based wave functions for the $f_{\frac{7}{2}}$ shell[9] to evaluate the separation densities for the valence neutrons of the calcium isotopes of even A. The results, which are shown in Fig. 1, illustrate a strong tendency for the two valence neutrons of ^{42}Ca to lie close together and a progressive weakening of the tendency as further neutron pairs are added to the same shell.

The dotted curve in Fig. 1 represents the separation density with the correlation term omitted and is the same for all four isotopes. The central values of the separation density exceed the uncorrelated value for ^{42}Ca and ^{44}Ca and are smaller than that value for ^{46}Ca and ^{48}Ca. Indeed, since

$$\rho_{sep}(0) = (1 + I\kappa_{N-s}) \int \rho^2(\mathbf{R}) d\mathbf{R} \ , \tag{22}$$

the central values of ρ_{sep} are in the ratios

$$\rho_{sep}^{^{42}Ca}(0) : \rho_{sep}^{^{44}Ca}(0) : \rho_{sep}^{^{46}Ca}(0) : \rho_{sep}^{^{48}Ca}(0) = 1 : \tfrac{1}{3} : \tfrac{1}{5} : \tfrac{1}{7} \ , \tag{23}$$

and the uncorrelated value is $\frac{1}{4}$ on the same scale.

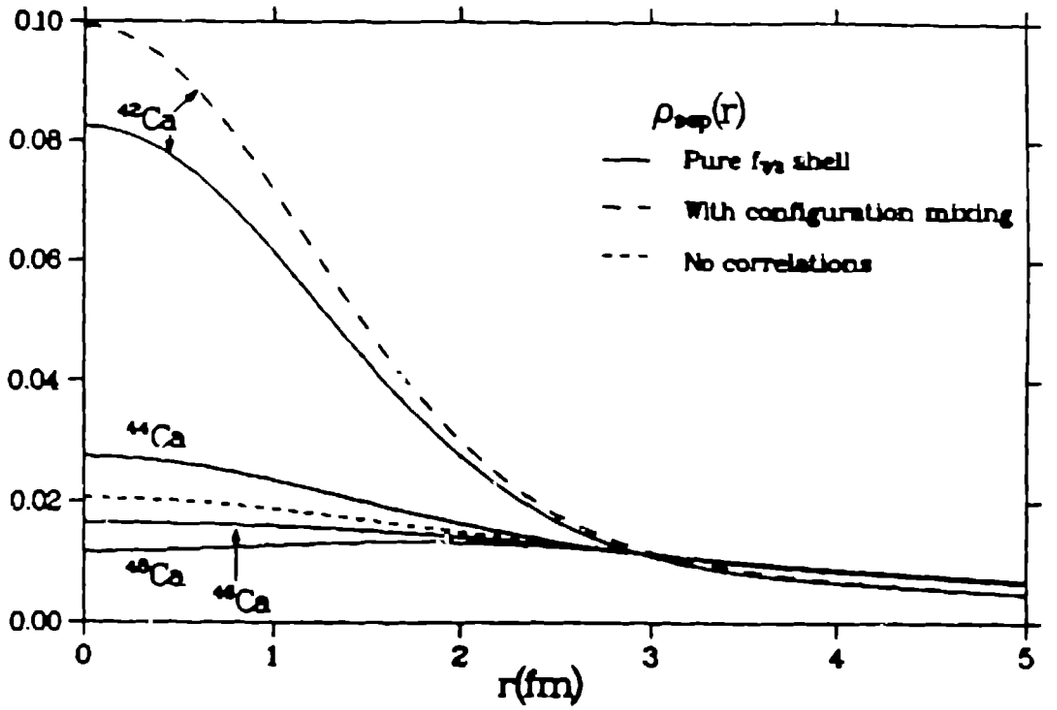

Fig. 1. The separation densities for ^{42}Ca, ^{44}Ca,^{46}Ca and ^{48}Ca (solid lines). The dotted curve is the separation density with the correlation function C_{N-s} set equal to zero. The dashed curve corresponds to a configuration-mixed representation of ^{42}Ca noted in the text.

The strong increase of the values of the separation density of valence neutrons in ^{42}Ca for small r results from large amount of overlap present in the spherically symmetric state of a pair of valence neutrons. Such an enhancement is common to other nuclei having two valence neutrons; we find it, however, to be weaker for lighter nuclei. As an example illustrating this effect, we present, in Fig. 2, the graphs of the separation densities for ^{14}C obtained with the correlated and uncorrelated two-neutron wave functions that are implicit in our previous

analysis.[1] Here the ratio of the central value of the separation densities to the corresponding uncorrelated value is approximately $\simeq 2$, which is significantly smaller than the value $\simeq 7$, we find for ^{42}Ca.

The fact that the double-charge-exchange cross sections are proportional, roughly speaking, to the squares of the values of the separation density, shows how dramatically the correlations affect the cross sections. If we take these ratios as a guide, we find the double-charge-exchange cross section for the single valence pair in ^{42}Ca to be some 49 times larger than that for any given pair in ^{48}Ca. The correlation effect in ^{42}Ca is thus so strong that its single pair yields a double-charge-exchange cross section significantly larger than the 28 pairs present in ^{48}Ca.

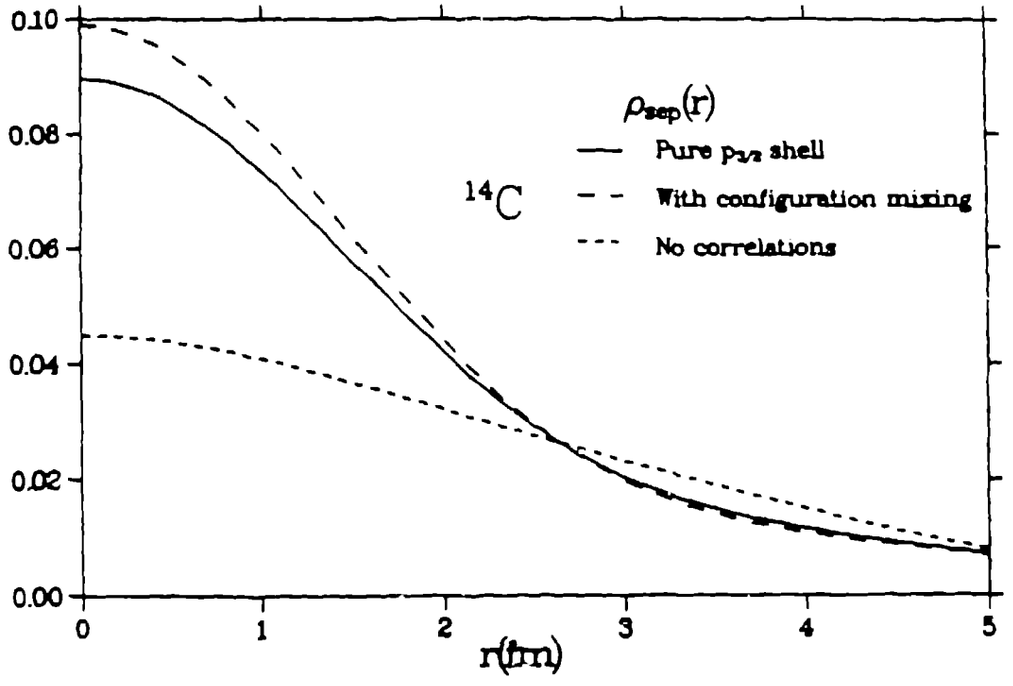

Fig. 2. Separation densities for valence neutrons in ^{14}C. The solid lines corresponds to the configuration-mixed representation of the two-neutron wave function.[11] The dashed curve represents the separation density obtained with the product of two $p_{3/2}$ single particle wave functions, i.e. with the correlation function set equal to zero.

In more accurate terms, the cross-sections are determined by the shapes as well as the central values of the separation density curves. The differential cross sections we have calculated by taking the momentum dependence of the πN charge-exchange amplitudes, together with their full spin dependence, into account and carrying out the integrations numerically are shown for the Ca isotopes at 35 MeV in Fig. 2. In this calculations the πN charge-exchange

scattering amplitudes entering in (4) have been parameterized in the familiar form

$$f(\mathbf{p}', \mathbf{p}) = a + b(\mathbf{p}' \cdot \mathbf{p}),$$
$$g(\mathbf{p}', \mathbf{p}) = c \ . \tag{24}$$

The coefficients a, b, and c appearing in the above formulas can obtained from the phase shift analysis of πN scattering. [12] The actual values of the parameters b and c, that our calculation was based on are identical to those following from ref. 12. The real part of the parameter a has been adjusted, similarly as in the analysis reported in ref. 1, by increasing it by 18% .

The double-charge-exchange cross-sections differ not only in their magnitudes, but in their angular dependences as well. The crosssection for ^{42}Ca is strongly forward-peaked and adding neutron pairs to the valence shell flattens it appreciably

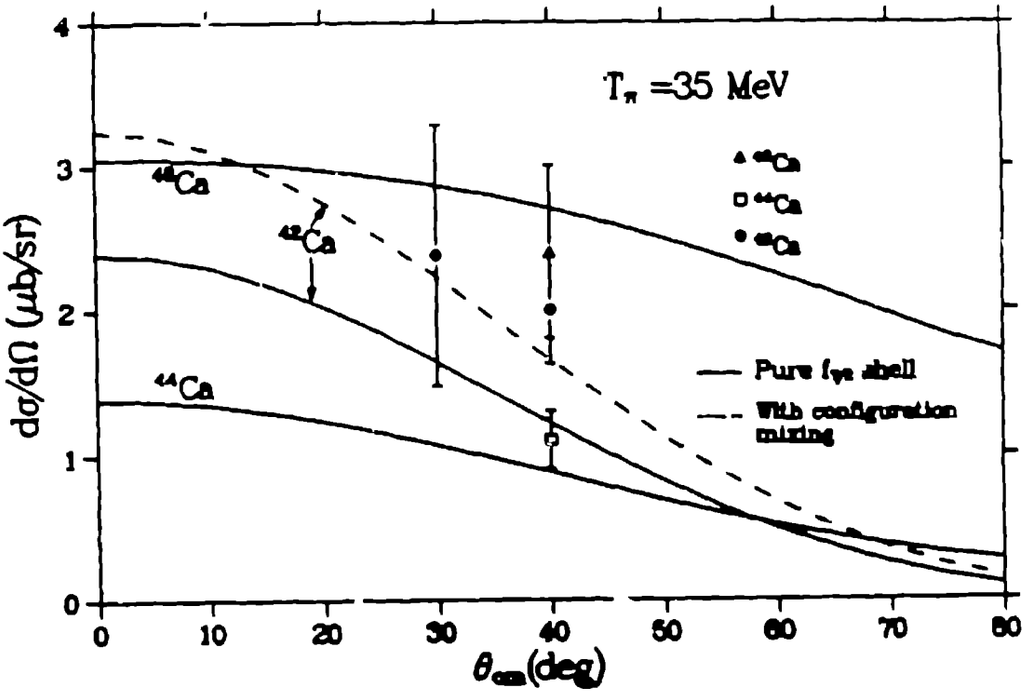

Fig. 3. Comparison between the theoretically predicted differential cross-sections and the experimental data for the reactions $\pi^+ {}^A\mathrm{Ca}(0^+) \rightarrow \pi^- {}^A\mathrm{Ti}(0^+)$, for A=42, 44 and 48, at 35 MeV. The data at 30° are from ref. 3, and those at 40° follow from ref. 4.

The reason for this behavior is that the angular distribution contributed by any neutron pair (m, n) is governed, roughly speaking, by the function

$$\left[S_{N-s}(\tfrac{1}{2}\mathbf{Q}, \tfrac{1}{2}\mathbf{Q})\right]^2 \tag{25}$$

representing the square of the form factor for the distribution

$$\rho_{cen}(R) = \int \rho^{(2)}_{N-s}(\mathbf{R} + \tfrac{1}{2}\mathbf{r}, \mathbf{R} - \tfrac{1}{2}\mathbf{r})d\mathbf{r} = \frac{1}{(2\pi)^3}\int e^{-i\mathbf{Q}\cdot\mathbf{R}} S_{N-s}(\tfrac{1}{2}\mathbf{Q}, \tfrac{1}{2}\mathbf{Q})d\mathbf{Q} \ , \tag{26}$$

of its centroid[1]

$$\mathbf{R} = \tfrac{1}{2}(\mathbf{r}_m + \mathbf{r}_n). \tag{27}$$

When the members of the pair are closely spaced, as in ^{42}Ca, the distribution of the centroid is similar to the distribution of the valence neutrons themselves. It extends to large radii and consequently has a sharply peaked form factor. For ^{48}Ca, on the other hand, the members of the valence pairs are generally much further apart. The distribution of their centroids tends therefore to be concentrated at smaller radii, and its form factor to be much less strongly peaked.

In the foregoing calculations we have used pure $f_{\frac{7}{2}}$ wave functions to describe the ground states of the Ca isotopes. Any configuration mixing present in these states may also affect the double-charge-exchange cross-sections, however, by changing both the correlated and uncorrelated parts of the two-particle density.

In order to verify the quantitative importance of the configuration mixing in the double-charge-exchange reaction in ^{42}Ca we have carried out a sample calculation for a mixed representation of the ground state of ^{42}Ca . In this calculation, we have assumed, the two neutron state to have 9% of the $(p_{\frac{3}{2}})^2$ configuration and 91% of $(f_{\frac{7}{2}})^2$ configuration.[13]

As is evident from the results presented in Fig. 1, the configuration mixing tends to increase the values of the separation density for small distances between the valence neutrons. It also results in an enhancement of the differential cross section, seen in Fig 3. The two measurements that have recently been reported[3,4] at 30° and 40° for the reaction $\pi^+ {}^{42}\mathrm{Ca}(0^+) \rightarrow \pi^- {}^{42}\mathrm{Ti}(0^+)$, are consistent within the stated errors with the prediction based on the configuration-mixed wave function. The measurement[3] that has been made at 40° for the reaction $\pi^+ {}^{48}\mathrm{Ca}(0^+) \rightarrow \pi^- {}^{48}\mathrm{Ti}(0^+)$ agrees with the calculation based on the pure $f_{\frac{7}{2}}$ valence neutron wave functions. The pure $f_{\frac{7}{2}}$ prediction for the differential cross-section for the reaction $\pi^+ {}^{44}\mathrm{Ca}(0^+) \rightarrow \pi^- {}^{44}\mathrm{Ti}(0^+)$ at 40° underestimates only slightly its experimental value.[3]

These measurements thus represent clear evidence for the detection of correlation effects in the valence shell.

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REFERENCES

[1] M. Bleszynski and R. Glauber, Phys. Rev. C **36**, 681 (1987).

[2] H. Baer, M. J. Leitch, R. L. Burman, M. D. Cooper, A. Z. Cui, B. J. Dropesky, G. C. Giesler, F. Irom, C. L. Morris, J. N. Knudson, J. R. Comfort, D. H. Wright, and R. Gilman, Phys. Rev. **C35**, 1425 (1987)

[3] Z. Weinfeld, E. Piasetzky, H. W. Baer, R. L. Burman, M. J. Leitch, C. L. Morris, D. H. Wright, S. H. Rokni and J. R. Comfort, Phys. Rev C, in press.

[4] K. K. Seth, in *Proceedings of the XI International Conference on Particles and Nuclei*, Kyoto, Japan, April 1987.

[5] R. J. Glauber, invited talk given at the *Nordic Meeting on Intermediate and High Energy Physics*, Geilo, Norway, January 1987.

[6] E. Bleszynski, M. Bleszynski, and R. Glauber, in *Proceedings of the XI International Conference on Particles and Nuclei*, Kyoto, Japan, April 1987, p 358.

[7] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777, (1935)

[8] J. Eisenberg and D. Koltun, *Theory of Meson Interactions with Nuclei* ,Wiley, New York, 1980.

[9] J. Eisenberg and W. Greiner, *Microsopic Theory of the Nucleus*, North Holland (1982).

[10] E. Bleszynski, *Nucleon-nucleon correlation effects in elastic and quasielastic scattering of hadrons from nuclei at high energies*, Ph.D. Thesis, University of Cracow, 1977, unpublished.

[11] S. Cohen and D. Kurath, Nucl. Phys. **73**, 1 (1965).

[12] R. Arndt, *Phase Shifts Analysis for πN Scattering*, Computer Program (1987).

[13] P. M. Endt and C. van der Leun, Nucl. Phys. **A310**, 1 (1978).